

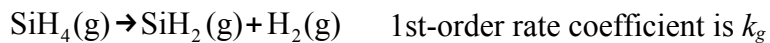
Modeling of polysilicon CVD in a wafer-stack tube reactor

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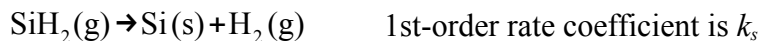
At some point during integrated circuit fabrication, you may want to deposit a layer of polycrystalline silicon over the wafer. This can be done by decomposition of silane, SiH₄, over wafer surfaces. Wafers are stacked in a quartz holder or "boat" with small separation between them and then the wafer stack is inserted into a heated quartz tube in which the reaction takes place.

The goal is a layer of polysilicon that has uniform thickness over the diameter of one wafer and from the first to the last wafer in the reactor. In order to do that, the reaction rate must be constant. The reaction rate will depend on gas composition and temperature. For now, assume that the temperature is constant everywhere in the reactor. We focus here on uniformity of gas composition and reaction rate. We follow the development in Middleman & Hochberg (1993), Chapter 12, in order to write equations that describe the changes in gas composition and reaction rate within the reactor.

The reactions are silane decomposition in the gas phase to silylene and hydrogen,



and the decomposition of silylene to produce solid silicon on the surface plus hydrogen gas.



Assume that the reverse reactions are negligibly slow under our conditions. This is a "series" reaction scheme. Also assume that gas density and velocity changes due to the change in gas moles is small such that we can solve equations for silane independently of equations for silylene.

The space between the wafer stack and the inner wall of the tube is the annulus. The volume of a small length of the annulus per unit length of annulus is,

$$\pi(R_t^2 - R_w^2) \Delta z / \Delta z = \pi(R_t^2 - R_w^2)$$

where R_t is the radius of the tube and R_w is the radius of a wafer. The area of the tube wall per unit length, and the geometric area of the wafer cylinder per unit length are, respectively,

$$2\pi R_t \Delta z / \Delta z = 2\pi R_t \quad ; \quad 2\pi R_w \Delta z / \Delta z = 2\pi R_w$$

Writing a balance on silane (subscript 1) in the annulus (subscript 0) under steady-state conditions gives us:

accumulation = net in by flow + generation by reaction in annulus + generation by diffusion out of wafer stack

$$0 = -\pi(R_t^2 - R_w^2)u \frac{\partial C_{10}}{\partial z} - \pi(R_t^2 - R_w^2)k_g C_{10} - 2\pi R_w D_1 \left[\frac{\partial C_1}{\partial r} \right]_{R_w}$$

$$u \frac{\partial C_{10}}{\partial z} + k_g C_{10} + \frac{2 R_w D_1}{(R_t^2 - R_w^2)} \left[\frac{\partial C_1}{\partial r} \right]_{R_w} = 0$$

where u (m/s) is the mean velocity of gas in the z direction (axial position) and the gas-phase reaction rate constant k_g has units (1/s).

The steady-state balance on silylene (subscript 2) in the annulus is :

accum = net in by flow + gen by reaction in annulus + gen by reaction on tube wall + gen by diffusion out of wafer stack

$$0 = -\pi (R_t^2 - R_w^2) u \frac{\partial C_{20}}{\partial z} + \pi (R_t^2 - R_w^2) k_g C_{10} - 2 \pi R_t k_s C_{20} - 2 \pi R_w D_2 \left[\frac{\partial C_2}{\partial r} \right]_{R_w}$$

$$u \frac{\partial C_{20}}{\partial z} - k_g C_{10} + \frac{2 R_t k_s C_{20}}{(R_t^2 - R_w^2)} + \frac{2 R_w D_2}{(R_t^2 - R_w^2)} \left[\frac{\partial C_2}{\partial r} \right]_{R_w} = 0$$

where the surface reaction rate coefficient k_s has units (m/s). Since the silane equation doesn't involve silylene, we can solve the silane equation first. Then, once we have the silane concentration in the reactor, we can solve for the silylene concentration and the silicon deposition rate.

Focus on the group just to the left of the equal sign of the silane balance:

$$u \frac{\partial C_{10}}{\partial z} + k_g C_{10} + \frac{2 R_w D_1}{(R_t^2 - R_w^2)} \left[\frac{\partial C_1}{\partial r} \right]_{R_w} = 0$$

In order to find the gradient in silane concentration between the wafers at the wafer edge, we need to write and solve a balance on silane in the region between a pair of wafers.

Assume that the gradient in silane concentration in the z direction between wafers at any radial position is small. The balance on silane over a ring-shaped control volume between two wafers gives, in dimensionless form,

$$\left(\frac{d^2 \psi}{d\lambda^2} + \frac{1}{\lambda} \frac{d\psi}{d\lambda} \right) - \phi^2 \psi = 0 \quad \text{B.C. at } \lambda=0 \quad \frac{d\psi}{d\lambda} = 0, \quad \text{and at } \lambda=1 \quad \psi=1$$

where $\psi = C_1(\lambda)/C_1(1) = C_1(\lambda)/C_{10}$, $\lambda = r/R_w$, $\phi = R_w(\text{m}) \left(\frac{k_g(1/\text{s})}{D_1(\text{m}^2/\text{s})} \right)^{1/2}$

The last term is a Thiele modulus, often called for gas-phase reactions (or in general) as a Damkoehler number. The solution for the dimensionless concentration profile is:

$$\psi(\lambda) = \left(\frac{I_0(\phi \lambda)}{I_0(\phi)} \right)$$

where I_0 is the modified (hyperbolic) Bessel function of the first kind and of order 0. In Matlab, this is function `besseli (order, argument)`. At WolframAlpha.com, this is `BesselI (order, argument)`.

In order to get uniform film deposition rate across the radius of the wafers we want a small value of Thiele modulus. The radius R_w is fixed by the need to make a lot of circuits per wafer. Operating at low pressure can increase the diffusivity D and, thus, decrease the Thiele modulus. LPCVD is Low Pressure CVD.

We can express the rate of diffusion of silane into the wafer stack in terms of an effectiveness factor, η .

Per unit per unit length of wafer stack, the rate of diffusion of silane into the gap between the wafers can be set equal to the effectiveness factor η times the rate of reaction of silane in the gap volume if the concentration of silane in the gap volume were to be everywhere equal to the silane concentration in the annulus. For a wafer thickness of d_w and a gap between wafers of d_g , the fraction of a unit length of wafer stack that is gap between wafers is $d_g/(d_w+d_g)$.

$$2\pi R_w \left(\frac{d_g}{d_g + d_w} \right) D_1 \left[\frac{\partial C_1}{\partial r} \right]_{R_w} = \pi R_w^2 \left(\frac{d_g}{d_g + d_w} \right) \eta k_g C_{10}$$

$$2\pi R_w D_1 \left[\frac{\partial C_1}{\partial r} \right]_{R_w} = \pi R_w^2 \eta k_g C_{10}$$

$$2 D_1 \left[\frac{\partial \psi}{\partial \lambda} \right]_{\lambda=1} = R_w^2 \eta k_g$$

$$\eta = \frac{2 D_1}{R_w^2 k_g} \left[\frac{\partial \psi}{\partial \lambda} \right]_{\lambda=1}$$

$$\eta = \frac{2}{\phi^2} \left[\frac{\partial \psi}{\partial \lambda} \right]_{\lambda=1}$$

$$\eta = \left(\frac{2}{\phi} \right) \left(\frac{I_1(\phi)}{I_0(\phi)} \right)$$

where I_1 is the modified Bessel function of the first kind and of order 1. Now the equation for silane in the annulus is:

$$u \frac{\partial C_{10}}{\partial z} + k_g C_{10} + \frac{2 R_w D_1}{(R_t^2 - R_w^2)} \left[\frac{\partial C_1}{\partial r} \right]_{R_w} = 0$$

$$u \frac{d C_{10}}{d z} + k_g C_{10} + \frac{R_w^2}{(R_t^2 - R_w^2)} \eta k_g C_{10} = 0$$

$$u \frac{d C_{10}}{d z} + \left[1 + \frac{\eta R_w^2}{(R_t^2 - R_w^2)} \right] k_g C_{10} = 0$$

$$\frac{d C_{10}}{d z} = - \left(\frac{1}{u} \right) \left[1 + \frac{\eta R_w^2}{(R_t^2 - R_w^2)} \right] k_g C_{10}$$

$$\frac{d C_{10}}{d z} = -k' C_{10} \quad \text{with initial condition } C_{10}(z=0) \text{ known}$$

Separating variables and integrating, we get the solution for the silane concentration in the annulus as a function of distance down the reactor.

$$C_{10}(z) = C_{10}(z=0) e^{-k' z}$$

At each distance z down the reactor, we have the silane concentration in the annulus. For constant rate coefficient and diffusion coefficient, the dimensionless concentration profile in the gap between wafers is constant with z .

$$\psi(\lambda) = \left(\frac{I_0(\phi \lambda)}{I_0(\phi)} \right)$$

However the actual values of silane concentration in the wafer gap will depend on the silane concentration in the annulus at the distance z .

Now that we can solve for the silane concentration at every location within the reactor, we can continue to solve for the silylene concentration and the silicon deposition rate. For the silylene balance, we need to evaluate the derivative of the silylene concentration between wafers at the wafer edges. So we need to solve for the silylene concentration profile between wafers.

Assume that the concentration of silane and silylene between wafers is approximately constant in the axial (z) direction but that these concentrations vary significantly in the radial (r) direction. To write a balance on silylene, the control volume is a ring-shaped volume between r and $r+\Delta r$.

accum = net in by diffusion + gen by reaction in gas + gen by reaction on wafer surfaces (2 surfaces per gap)

$$0 = \left[-D_2 2\pi r d_g \frac{d C_2}{d r} \right]_r - \left[-D_2 2\pi r d_g \frac{d C_2}{d r} \right]_{r+\Delta r} + k_g C_1 2\pi r \Delta r d_g - k_s C_2 4\pi r \Delta r$$

The area of the wafer surface on one the side of the control volume is

$$\pi (r + \Delta r)^2 - \pi r^2 = \pi (r^2 + 2r \Delta r + \Delta r^2 - r^2) \approx 2\pi r \Delta r$$

Thus, the volume of the control volume is $2\pi r \Delta r d_g$ and the area of two sides is $4\pi r \Delta r$

Divide by $2\pi r \Delta r d_g$ remember that the r in the diffusion terms in brackets is a function of r , and take the limit as Δr approaches zero:

$$\frac{D_2}{r} \frac{d}{dr} \left[r \frac{dC_2}{dr} \right] + k_g C_1 - \left(\frac{2k_s}{d_g} \right) C_2 = 0 \quad \text{with B.C. at } r=0, \frac{dC_2}{dr} = 0 ; \text{ at } r=R_w, C_2 = C_{20}$$

We have $C_1(r)$ from above. Middleman and Hochberg give the solution as

$$C_2(\lambda) = \left[C_{20} + \frac{C_{10}}{1-\varphi^2} \right] \left[\frac{I_0(\Phi_s \lambda)}{I_0(\Phi_s)} \right] + \left[\frac{C_{10}}{\varphi^2-1} \right] \left[\frac{I_0(\Phi_g \lambda)}{I_0(\Phi_g)} \right]$$

where

$$\lambda = \frac{r}{R_w} ; \quad \Phi_s^2 = \frac{2R_w^2 k_s}{d_g D_2} ; \quad \Phi_g^2 = \frac{R_w^2 k_g}{D_1} ; \quad \varphi^2 = \frac{\Phi_s^2}{\Phi_g^2} = \frac{2k_s D_1}{d_g k_g D_2}$$

Now, for the silylene balance,

$$u \frac{\partial C_{20}}{\partial z} - k_g C_{10} + \frac{2R_t k_s C_{20}}{(R_t^2 - R_w^2)} + \frac{2R_w D_2}{(R_t^2 - R_w^2)} \left[\frac{\partial C_2}{\partial r} \right]_{R_w} = 0$$

we can evaluate the gradient in silylene at the wafer edge:

$$R_w \left[\frac{\partial C_2}{\partial r} \right]_{R_w} = \left[\frac{\partial C_2}{\partial \lambda} \right]_{\lambda=1} = \left[C_{20} + \frac{C_{10}}{1-\varphi^2} \right] \left[\frac{\Phi_s I_1(\Phi_s)}{I_0(\Phi_s)} \right] + \left[\frac{C_{10}}{\varphi^2-1} \right] \left[\frac{\Phi_g I_1(\Phi_g)}{I_0(\Phi_g)} \right]$$

The silylene balance becomes an ODE that is coupled to the solution for $C_{10}(z)$ that we obtained above.

Middleman and Hochberg consider a limiting case and propose that uniform deposition across wafers can be obtained for that case by adjusting the tube radius R_t and the wafer spacing d_g given a specified wafer radius R_w .

The limiting case is where both (1) the concentration of silane between wafers is relatively constant with radius and (2) the convection term in the silylene balance is relatively small.

In that case, they propose that the silylene concentration and, thus, the silicon deposition rate, is constant with radius when

$$\frac{d_g}{R_w} = \frac{R_t}{R_w} - \frac{R_w}{R_t} \quad \text{where } R_t > R_w$$

$$\frac{d_g}{R_w} = 0.19 \text{ for } \frac{R_t}{R_w} = 1.10 \quad ; \quad \frac{d_g}{R_w} = 0.098 \text{ for } \frac{R_t}{R_w} = 1.05 \quad ; \quad \frac{d_g}{R_w} = 0.020 \text{ for } \frac{R_t}{R_w} = 1.01$$

I suspect that this condition results in a relatively large gap between wafers. For 300 mm-diameter wafers in a 315 mm-diameter tube ($R_t/R_w = 1.05$), the gap would be 14.7 mm. Photos of wafer stacks appear to show the wafers closer together than that.

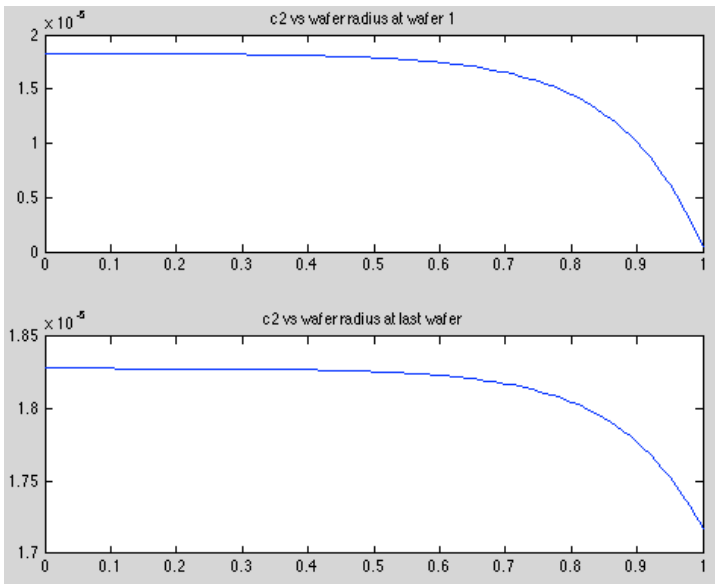
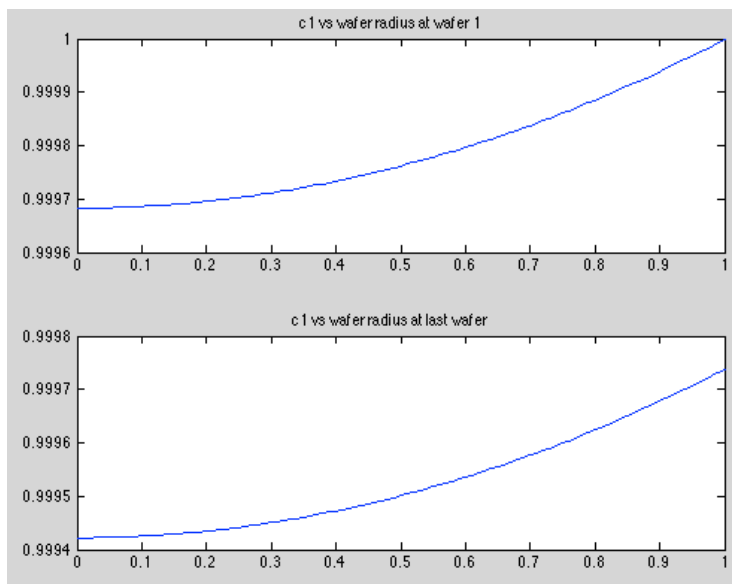
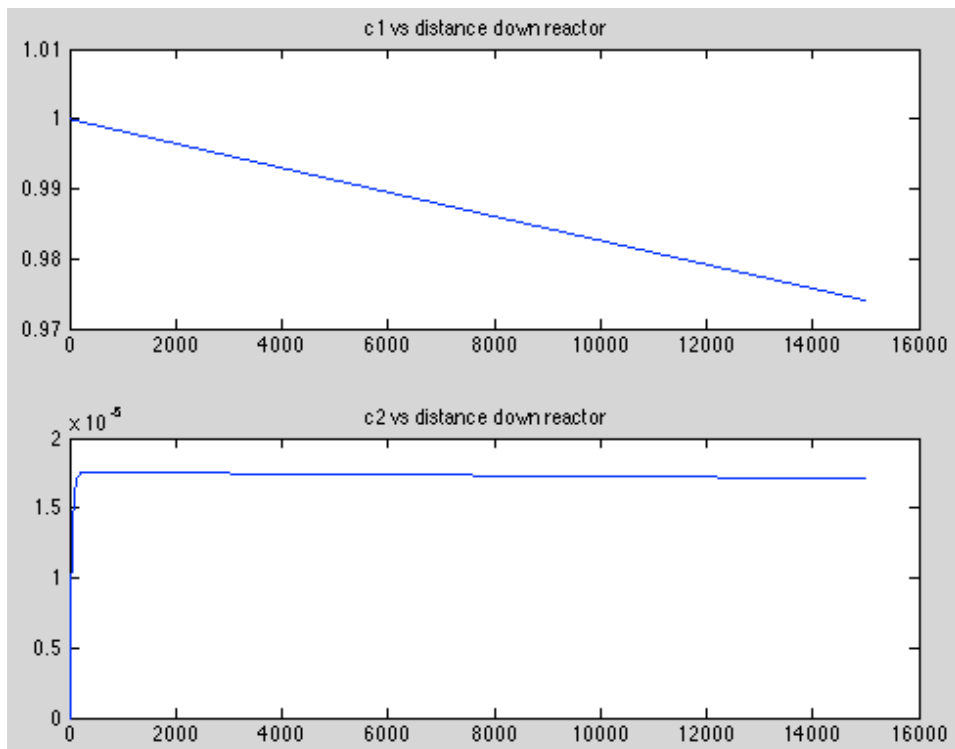
This condition on reactor geometry maintains a constant surface-to-volume ratio of $(2/d_g)$ when $d_g \gg d_w$. The gap between wafers d_g is on the order of 0.01 to 0.001 m, so the surface-to-volume ratio is one the order of 100 to 1000 m^2/m^3 for this condition.

Why may this condition maintain a constant silylene profile across a wafer? A constant surface-to-volume ratio implies a constant ratio of gas-phase generation of silylene to surface deposition of silicon by silylene.

We wrote a Matlab script that solves this system of equations by numerical approximation with the addition of a reaction step that deposits Si(s) directly from silane. The Matlab script is listed at the bottom of this document. The new reaction was added to the Thiele modulus ϕ and to the silane balance in the annulus. Rate constants were estimated from the data of Tao (1993) and Weerts et al. (1997). For the two reactions that deposit silicon solid, refer to Table 4 of Tao (1993) to estimate rate coefficients. Table 4 shows decomposition probabilities of SiH_4 and SiH_2 at various temperatures. Assume that these are the probabilities that a reaction occurs when a gas-phase molecule collides with the surface. In order to convert these data into reaction rate coefficients, you will also need to compute the "collision frequency" at reactor conditions, i.e., the rate at which gas molecules collide with the surface. See my notes on "Step coverage and determination of sticking coefficients" or other references about collision frequencies.

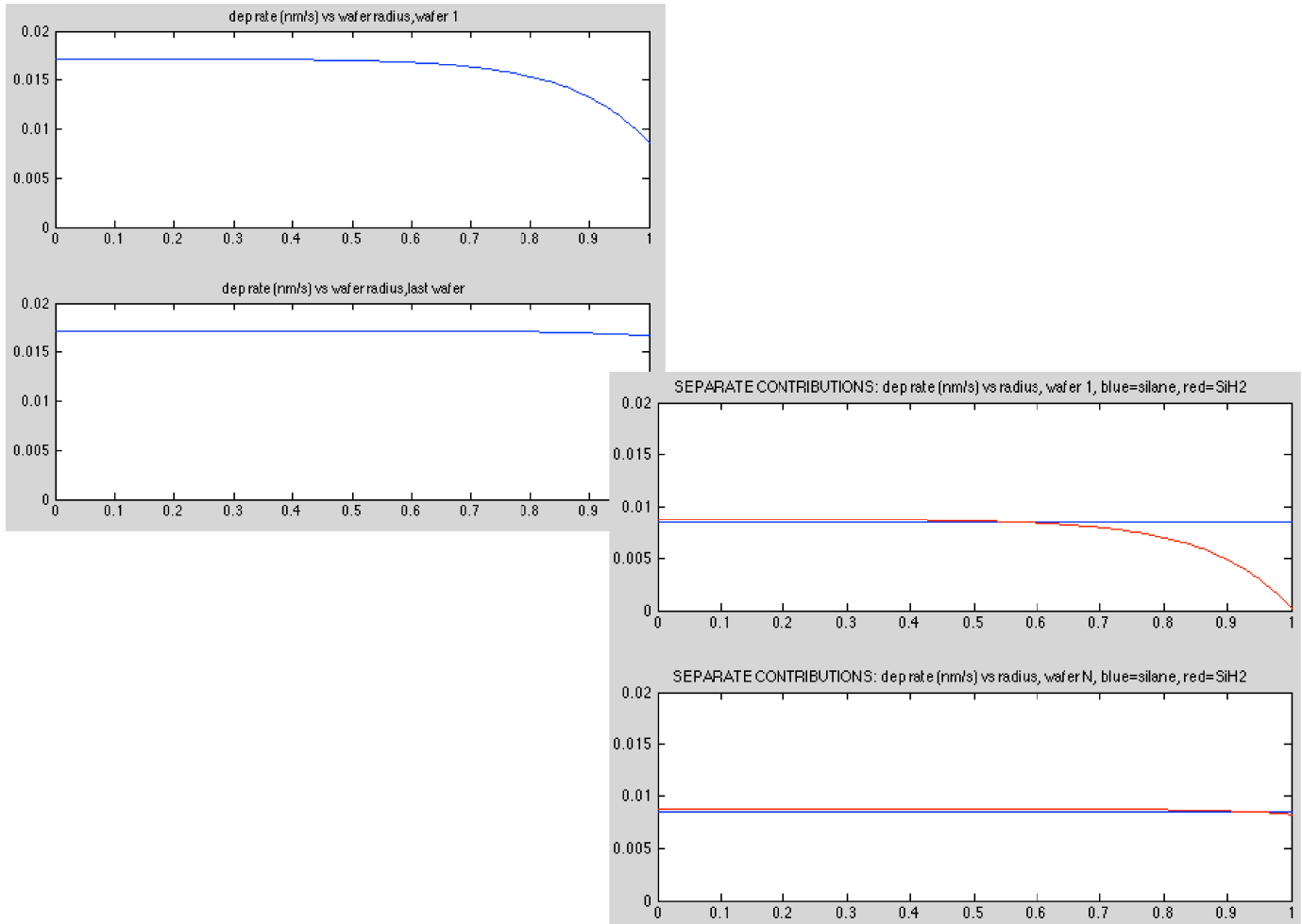
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Below on this page and the next page are my plots. The x-axis scale in the first plot is integration steps, here 100 per wafer for 150 wafers. The concentrations c_1 and c_2 are dimensionless, being normalized to the inlet silane concentration.



The deposition rate appears to be pretty uniform in the reactor.

The actual inlet silane concentration must be used to compute Si(s) deposition rates. I used the molar volume of Si(s) from a NIST web page to convert deposition rates from (mol/m²/s) to (nm/s). I got a maximum deposition rate of about 0.02 nm/s. Weerts, et al. had a maximum rate of about 0.1 nm/s, so I'm within an order of magnitude at least...



REFERENCES

Middleman, S., Hochberg, A., 1993. Process Engineering: Analysis in Semiconductor Device Fabrication, First Edition edition. McGraw-Hill College, New York.

Tao, M., 1993. "Growth kinetics and reaction mechanism of silicon chemical vapour deposition from silane," *Thin Solid Films*, vol. 223, no. 2, pp. 201–21.

<<http://www.sciencedirect.com/science/article/pii/004060909390522Q#>> See PDF download link in top-left of page under "Science Direct" (a little hard to find).

Weerts, W. L. M. , de Croon, M. H. J., and Marin, G. B., 1997. "Low-Pressure Chemical Vapor Deposition of Polycrystalline Silicon: Analysis of Nonuniform Growth in an Industrial-Scale Reactor," *J. Electrochem. Soc.*, vol. 144, no. 9, pp. 3213-3221.

<<http://alexandria.tue.nl/repository/freearticles/617790.pdf>>

KEY VALUES COMPUTED BY PROGRAM

Q = 0.0891 m³/s, inlet volumetric flow rate of silane at reaction conditions
u = 1.8171 m/s, inlet linear velocity of gas in annulus
kg = 0.0038 1/s, gas-phase silane > SiH₂ rate coefficient
PsilaneTrxr = 1.0826e-06, silane decomposition Probability upon surface collision
PSiH2Trxr = 0.0619, SiH₂ surface decomposition Probability upon surface collision
ks1 = 2.0857e-04 m/s, surface reaction rate coefficient, silane decomposition on surface
ks = 11.9280 m/s, surface reaction rate coefficient, SiH₂ decomposition on surface
Fs = 8.3027, Thiele modulus between wafers, SiH₂ decomposition on surface
Fg = 0.0355, Thiele modulus, silane to SiH₂ in gas + new silane decomposition
maxR = 0.0172 nm/s, maximum film growth rate in reactor

LISTING OF MAIN MATLAB PROGRAM

```
% By R. K. Herz, rherz@ucsd.edu (2014)
% Si CVD in LPCVD wafer stack reactor
% SiH4 reacts to solid film and to SiH2
% SiH2 reacts to solid film
% solid deposition rate coefficients from Tao (1993)
% gas phase rate coefficient for silane to SiH2 from Weerts, et al. (1997)
% M. Tao, Thin Solid Films, vol. 223, no. 2, p. 201, Feb. 1993.
% W. Weerts, et al., J. Electrochem. Soc., vol 144, p. 3213, Sept 1997
clear

% Conditions and data from Weerts et al.
Trxr = 900; % K
P = 25; % Pa, pressure
LL = 0.75 % m, reactor length, compare to L below
Rt = 0.16; % m, tube radius
Rw = 0.10; % m, wafer radius
dg = 0.00503; % m, wafer spacing
Nwafer = 150; % number of wafers
D2 = 0.688; % m2/s, diffusivity of silylene in H2
D1 = D2; % m2/s, assume same diffusivity of silane in H2
Q = 400; % sccm = cm3/min at 1 atm, 20 C
Q = Q * (1/1003)*(1/60)*(Trxr/273)*(101.3e3/P) % m3/s at reactor T & P
Ax = pi*(Rt2-Rw2); % m2, cross-sectional area of annulus
u = Q/Ax % m/s, mean linear velocity of gas in annulus at inlet

% homog reaction of silane to SiH2
% get rate coeffic from Weerts, et al.
A = 1.28e10; % (1/s), pre-exponential factor
E = 215.8e3; % J/mol, activation energy
Rg = 8.3145; % J/K/mol, also (m3 Pa)/K/mol, ideal gas constant
kg = A*exp(-E/Rg/Trxr) % (1/s)

% inlet gas conc - assume pure SiH4
% Cin = n/V = P/Rg/T
Cin = P/Rg/Trxr; % mol/m3, inlet gas conc

% Decomposition probabilities from Tao
% increase with T, check Arrhenius dependence
T = 273 + [500:100:1100]; % K
Psilane = [6.9e-8 5.8e-7 7.1e-6 1.2e-5 3.6e-5 9.2e-5 2.0e-4];
invT = 1./T;
coef = polyfit(invT,log(Psilane),1); % fit to 1st order polynomial
PsilaneLNX = polyval(coef,invT);
plot (invT,log(Psilane),'bo',invT,PsilaneLNX,'rx') % check fit
title('ln(silane decomp prob) vs. 1/T')
PsilaneTrxr = exp(polyval(coef,1/Trxr)) % get value at T
% pause
% repeat for PSiH2
PSiH2 = [3.4e-2 5.6e-2 8.2e-2 1.1e-1 1.4e-1 1.8e-1 2.1e-1];
coef = polyfit(invT,log(PSiH2),1); % fit to 1st order polynomial
PSiH2LNX = polyval(coef,invT);
plot (invT,log(PSiH2),'bo',invT,PSiH2LNX,'rx') % check fit
title('ln(SiH2 decomp prob) vs. 1/T')
```

```

PSiH2Trxr = exp(polyval(coef,1/Trxr)) % get value at T
%pause

% so now we have decomposition probabilities at Trxr from Tao
% PsilaneTrxr and PSiH2Trxr

% get surface rate coefficient ks from Komiyama, et al. Eqn(6d)
% mean velocity of gas molecules from kinetic theory of gases
v = sqrt(8*Rg*Trxr/pi/M) in m/s, where M is mol wt of SiH4
Rg = 8.3145; % (m3 Pa)/K/mol, J/K/mol, ideal gas constant
M = 32.09e-3; % kg/mol, molecular weight of SiH4 = 28.09 + 4
v = sqrt(8*Rg*Trxr/pi/M) % m/s, mean velocity gas molecules
ks1 = v*PsilaneTrxr/4 % m/s, surface reaction rate coefficient for silane
ks = v*PSiH2Trxr/4 % m/s, surface reaction rate coefficient for SiH2

% with gas conc in mol/m3, the surface rate coefficients
% give rates in mol/m2/s and the homog rate coeff gives rates in mol/m3/s
% we ignore change in gas moles with reaction

% SV is ratio of two wafer surfaces to volume between two wafers
SV = 2/dg; % (1/m), SV = (2*pi*Rw^2)/(pi*Rw^2*dg)
L = (Nwafer-1) * dg % m, length of wafer stack for "thin" wafers
% compare L to LL above from Weerts

Fs = Rw*sqrt(ks*SV/D2) % d'less, Thiele modulus-s
Fg = Rw*sqrt((kg+ks1*SV)/D1) % d'less, *NEW* ks1*SV for SiH4 > solid

L1 = 2*Rt*ks/(Rt^2 - Rw^2); % 1/s
L11 = 2*Rt*ks1/(Rt^2 - Rw^2); % 1/s, *NEW* ks1 for SiH4 > solid
L21 = 2*D1/(Rt^2 - Rw^2); % 1/s
L22 = 2*D2/(Rt^2 - Rw^2); % 1/s
uu = L/u; % s, space time of gas in annulus

% Use simple Euler integration of initial value problem, 2 coupled ODE's
% Alternatively, the c1 equation has an analytical solution

% set initial conditions for concentrations down reactor
c1(1) = 1; % dimensionless, inlet conc of silane
c2(1) = 0; % dimensionless, inlet conc of silylene

% need small steps at inlet because of rapid increase in c2 conc
% for that reason, take smaller steps than wafer increment
% alternatively could use variable step size and other more
% sophisticated integration algorithms or MATLAB function ode45
lam = 0; % dimensionless reactor length
n = 1; % array index
dlam = 1/Nwafer/100;

while lam < 1

    % Compute deriv of c1 and c2 with respect to wafer radius at wafer edge
    % s = r/Rw is dimensionless wafer radius

    % use analytical solution for c1 deriv
    dc1ds = c1(n)*Fg*besseli(1,Fg)/besseli(0,Fg);
    % use simple numerical difference approximation for c2 deriv
    % Middleman & Hochberg supply analytical solution but this also works
    dc2ds = (waferinter2(c1(n),c2(n),Fs,Fg,1) ...
        - waferinter2(c1(n),c2(n),Fs,Fg,0.99))/0.01;

    % compute deriv of c1 and c2 with respect to axial distance lambda
    dc1dlam = -uu*(kg*c1(n) + L11*c1(n) + L21*dc1ds);
    dc2dlam = -uu*(-kg*c1(n) + L1*c2(n) + L21*dc2ds);

    % compute c1 and c2 at next step down stack
    c1(n+1) = c1(n) + dc1dlam * dlam;
    c2(n+1) = c2(n) + dc2dlam * dlam;

    % increment length and array index for new loop control
    lam = lam + dlam;
    n = n + 1;
end

```

```

figure(1)
subplot(2,1,1), plot(c1), title('c1 vs distance down reactor')
subplot(2,1,2), plot(c2), title('c2 vs distance down reactor')

s = 0:0.01:1;

figure(2)
c2i2 = waferinter2(c1(2),c2(2),Fs,Fg,s);
c2iN = waferinter2(c1(Nwafer+1),c2(Nwafer+1),Fs,Fg,s);
subplot(2,1,1), plot(s,c2i2), title('c2 vs wafer radius at wafer 1')
subplot(2,1,2), plot(s,c2iN), title('c2 vs wafer radius at last wafer')

figure(3)
s = 0:0.01:1;
cli2 = waferinter1(c1(2),Fg,s);
cliN = waferinter1(c1(Nwafer+1),Fg,s);
subplot(2,1,1), plot(s,cli2), title('c1 vs wafer radius at wafer 1')
subplot(2,1,2), plot(s,cliN), title('c1 vs wafer radius at last wafer')

% Si(s) deposition rate units are mol/m2/s
depRwafer2 = Cin*(ks*c2i2 + ks1*cli2);
depRwaferN = Cin*(ks*c2iN + ks1*cliN);
% get max rate in reactor in (mol/m2/s) and convert to (nm/s)
% molar volume of Si(s) = 1.2059e-5 m3/mol per NIST on web
% 1 (mol/m2/s)*(1.2059e-5 m3/mol)*(1 nm)/(1e-9 m) = 1.2059e4 nm/s
fnm = 1.2059e4; % factor to multiply rate in (mol/m2/s) to get (nm/s)
depRwafer2 = depRwafer2 * fnm; % nm/s
depRwaferN = depRwaferN * fnm; % nm/s
maxR2 = max(depRwafer2);
maxRN = max(depRwaferN);
maxR = max([maxR2 maxRN]) % nm/s

figure(4)
subplot(2,1,1), plot(s,depRwafer2), title('dep rate (nm/s) vs wafer radius,wafer 1')
axis([0 1 0 0.02])
subplot(2,1,2), plot(s,depRwaferN), title('dep rate (nm/s) vs wafer radius,last wafer')
axis([0 1 0 0.02])

depR1wafer2 = Cin*ks1*cli2 * fnm;
depR2wafer2 = Cin*ks*c2i2 * fnm;
figure(5)
subplot(2,1,1), plot(s,depR1wafer2,'b',s,depR2wafer2,'r')
title('SEPARATE CONTRIBUTIONS: dep rate (nm/s) vs radius, wafer 1, blue=silane, red=SiH2')
axis([0 1 0 0.02])

depR1waferN = Cin*ks1*cliN * fnm;
depR2waferN = Cin*ks*c2iN * fnm;
subplot(2,1,2), plot(s,depR1waferN,'b',s,depR2waferN,'r')
title('SEPARATE CONTRIBUTIONS: dep rate (nm/s) vs radius, wafer N, blue=silane, red=SiH2')
axis([0 1 0 0.02])

```

LISTING OF FUNCTION FILE waferinter1.m

```

function c1 = waferinter1(c10,Fg,s)
% Middleman & Hochberg Eqn (12-194)

c1 = c10 * besseli(0,Fg*s)/besseli(0,Fg);

```

LISTING OF FUNCTION FILE waferinter2.m

```

function c2 = waferinter2(c10,c20,Fs,Fg,s)
% Middleman & Hochberg Eqn (12-197)

phi = Fs/Fg;
c2 = (c20 + c10/(1-phi^2))*besseli(0,Fs*s)/besseli(0,Fs) ...
+ c10/(phi^2-1)*besseli(0,Fg*s)/besseli(0,Fg);

```