Main objectives:

1) Learn to write math models of some dynamic processes.

2) Learn to design a controller, a device that can maintain a process at a desired state while the process is subject to disturbances.

Simple examples:

– Maintain liquid level in a tank at a desired value as the inlet flow rate changes due to changes in upstream processes.

– Maintain liquid temperature in a tank at a desired value as the temperature of the inlet liquid changes due to changes in upstream processes.

Examples in your daily life:

– Home heater and thermostat.

– Control of air-fuel ratio in automobile engine.

What other examples come to mind?

Why do we need to control a process?

In other classes, we learn to design a steady-state process that operates under optimal, or at least good, conditions.

When designing our process, we assume that inputs remain constant.

In "real life," however, nothing remains perfectly constant. In fact, inputs may change substantially.

Our system won't be completely isolated from its surroundings.

We might be able to determine what happens in our process, but we don't have much say in what goes on in the surroundings.

For example, someone may open a valve unexpectedly in a pipe that feeds into our process. We want to make sure our process doesn't overflow.

The general idea of process control:

We choose a setpoint or reference which is a value that we want one of the variables in our system to have. This variable might be the height of liquid in a tank. This variable is the controlled variable.

We measure the controlled variable and compare its value to the setpoint. The difference in these values is the error. Usually, the error equals the setpoint value minus the measured value of the controlled variable.

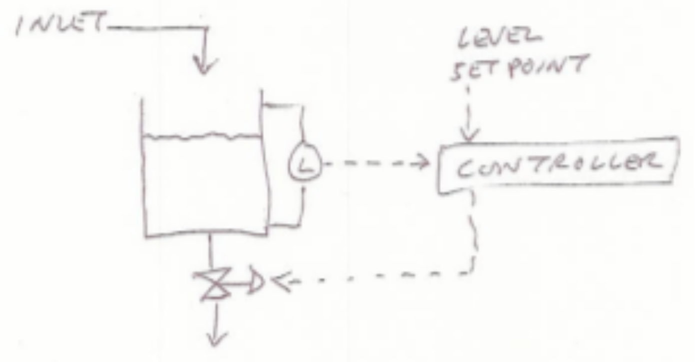
An error can be present when one of the inputs to the system changes to provide a disturbance or load. We will also get an error when we change the setpoint.

We feed the error signal to the controller we designed. The controller does some "thinking" – executes its control algorithm – and then puts out a control signal or command signal.

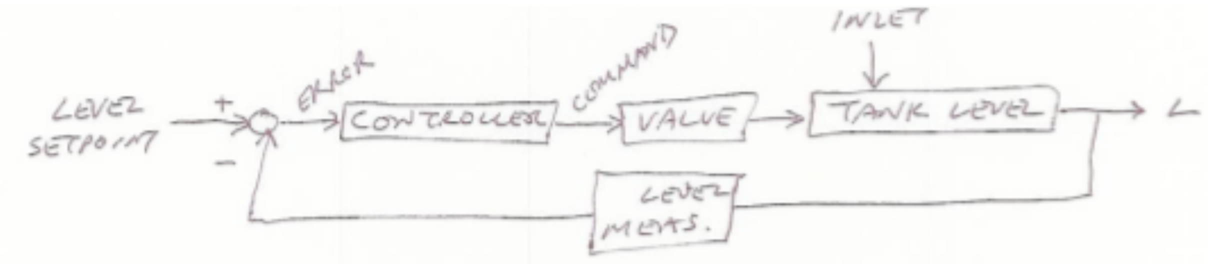
The command signal is sent to an actuator that changes the value of a process variable that the manipulated variable. An example of an actuator is a valve on the outlet pipe of a tank, where the opening of the valve can be changed by the command signal. The manipulated variable in this case is the outlet flow rate of liquid from the tank.

If all goes well, when the level in the tank goes above the setpoint level, the controller opens the outlet valve and the level drops back towards the setpoint level.

Schematic diagram showing the physical layout:



Block diagram of control system showing information flow:



This type of control system is called "feedback" control.

There are also "feedforward" controllers.

In a feedforward controller, we measure the value of an input variable and compare its current value to a reference value. When the values differ, we know that the change in the input is going to move our controlled variable away from the setpoint. So our feedforward controller takes some action, e.g., sends a command to the outlet control valve, in order to offset the effect of the change in the input on the controlled variable.

In feedback control, we measure the value of the controlled variable and then take action in order to keep the controlled variable at the setpoint.

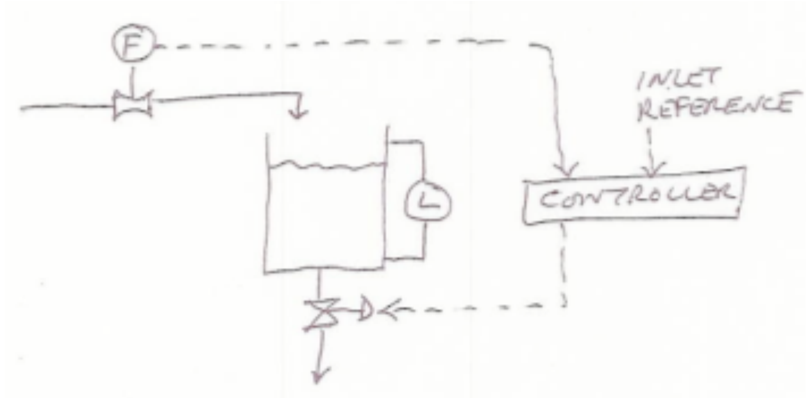
In feedforward control, we measure the value of an input variable and then take action in order to try keep the controlled variable at the setpoint. The controller action must be based on a mathematical model of how the system responds to the disturbance.

Feedback control is more common. It can handle a change in any of the inputs to the system.

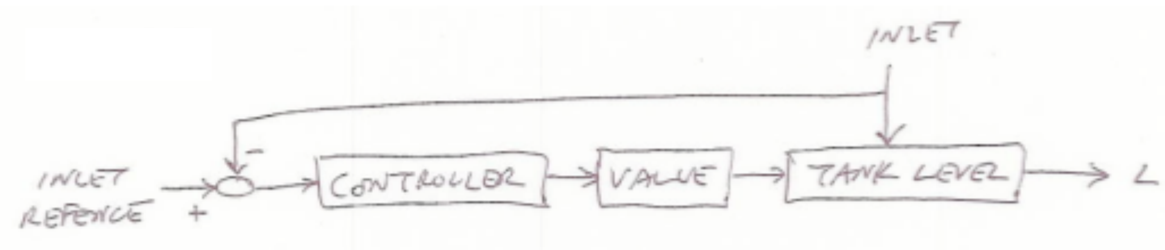
Feedforward control can only try to handle a change in the input variable it is measuring. Its value is that it can "anticipate" the effect of a disturbance so as to provide a faster response.

Usually, whenever a feedforward controller is used, a feedback controller is also present. This is because our math model of the system won't be perfect such that feedforward control by itself can't keep the controlled variable exactly at its set point. Also, feedforward control on one disturbance can't compensate for other disturbances.

Schematic diagram showing the physical layout in "feedfoward" control:



Block diagram of control system showing information flow:



Remember that a feedback controller will usually be added to this system in order to control other disturbances.

One thing you might have observed so far – there are a lot of terms involved in process control.

So far, the terms we have encountered include:

controlled variable
measured variable
manipulated variable
setpoint
reference
disturbance
load
error
control algorithm
command signal
control signal
acutator
schematic diagram
block diagram
feedback
feedforward

You need to learn all of the terms in the textbook – recognize them, understand them, be able to give a definition and an example.

Earlier, we mentioned the we will need to write math models of processes.

In chemical process control, we will usually write one or more of the following:

- total mass balances
- element balances
- component balances
- energy balances

Most of the time in CENG 100, we considered steady-state systems where the accumulation term was zero.

In CENG 120, we are interested in unsteady-state systems, also termed dynamic systems, and transient systems. Therefore, the accumulation term will be non zero.

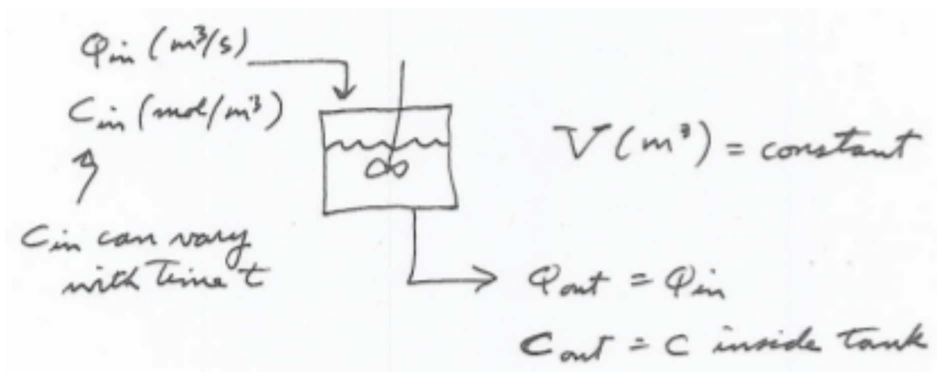
We are also interested in following our systems continuously in time, not just over a finite time interval. Therefore, the accumulation term will be a time derivative.

Here is a simple process, a tank containing a constant volume of liquid and with constant inlet and outlet flow rates of liquid.

A chemical component is present in the inlet flow to the tank. The concentration of this component can change with time without changing the flow rates of liquid.

As the inlet concentration changes, the concentration in the tank will change.

The tank is well-mixed such that the concentration in the outlet pipe is the same as that everywhere inside the tank.



Now write a component balance around the volume of liquid in the tank:

COMPONENT BALANCE

ACCUM = IN − OUT + GEN

$$V \frac{dC}{dt} = \varphi C_{in} - \varphi C_{out} + 0 \text{ (NO RXN)}$$

$$m^3 \frac{mol}{m^3 \cdot s} \qquad \frac{m^3}{s} \frac{mol}{m^3}$$

Rearranging,

$$\tau \frac{dC}{dt} + C = C_{in}(t)$$

$C_{in}(t)$ → FORCING FUNCTION, INPUT

where $\tau \equiv \frac{V}{\varphi}$ & τ has units of sec.

where we have defined the space time τ. The right hand side of the equation is the forcing function or input.

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In other fields, control engineers may also write

– force balances (mechanical and structural engineering)
– current balances (electrical engineering)

After we write a math model of our process, we will often have some unknown values in the model. An example would be the flow coefficient of a control valve. We can do experiments and and analyze the data in order to determine the unknown values.

After we write a math model of our process coupled to our controller, we need to determine values associated with the controller in order to obtain good control.

So we are going to do quite a bit of math.

For this first course in control, we are going to try to make things easy for you ;)

We are going to consider linear systems.

A linear system is one that can be modeled with linear differential equations. A linear differential equation does not contain multiplicative products of variables with themselves or other variables. That is, not x^2 nor xy where x and y are variables. It can contain second and higher derivatives.

Furthermore, we are going to consider linear systems with constant coefficients. That is, in a term such as a_1x, the value of the coefficient a_1 remains constant in time.

In the mixing tank model a couple slides ago, the space time (tau) was specified to be constant.

Finally, we are going to consider lumped parameter systems. In a lumped parameter system, a variable has a value that changes in time but not in space. Everything in the process is "lumped together in space."

We will only have derivatives with respect to time and, thus, ordinary differential equations.

An example is a well-mixed tank. The temperature of the liquid in the tank is the same everywhere in the tank.

The alternative is a distributed parameter system. In such a system, the value of a variable can vary in both space and time.

You will have derivatives in space as well as time, so you will have partial differential equations.

An example is liquid flowing through a pipe. The temperature of the liquid can vary with position down the pipe as well as with time.

Are you worried that we are making things too simple for you?

Don't worry. Even linear systems can exhibit complex behavior.

There is a little problem. No real system is perfectly linear.

Some systems may approach linear behavior closely enough such that a linear analysis will give an answer that is good enough for our purposes. In the mixing tank a couple slides ago, we specified that the tank was sufficiently well mixed such that we could assume that the component concentration was the same everywhere in the tank. A real mixing tank could only approach this condition.

Other systems may be strongly nonlinear. What to do for nonlinear systems?

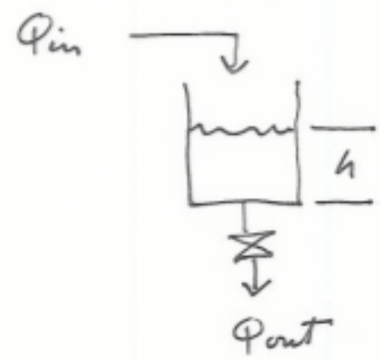
We will learn to linearize the system.

We will consider relatively small deviations of variable values about some steady-state condition.

In our math model, we will use a first-order Taylor approximation of the response of our system (accumulation term in balance equation) to changes in the input variables from their steady-state values (in – out + generation terms).

Using a first-order Taylor approximation results in linear terms.

Consider liquid flowing into and out of a tank. The inlet flow rate can vary with time. The outlet flow rate is proportional to the square root of the hydrostatic head. Thus, the outlet flow rate may be different than the inlet flow rate, causing the height of liquid in the tank to vary with time.



TOTAL MASS BAL

$$\rho \frac{dV}{dt} = \rho q_{in} - \rho q_{out}$$

where $q_{out} = C_v \left(\rho \left(\frac{g}{g_c} \right) h \right)^{1/2}$

$$V = Ah$$

$$A\frac{dh}{dt} = \Phi_{in}(t) - \underbrace{c_v\left(\rho\frac{g}{g_c}\right)^{1/2}}_{\beta} h^{1/2}$$

The equation is nonlinear in the liquid height, h. Now apply the first-order Taylor approximation to the $h^{1/2}$ term about an initial steady-state value of the height, h_s.

$$f(h) = h^{1/2} \simeq h_s^{1/2} + \left.\frac{\partial f}{\partial h}\right|_s (h - h_s)$$

$$\simeq h_s^{1/2} + \left(\frac{1}{2}h_s^{-1/2}\right)(h - h_s)$$

Remember that the initial steady-state value of h, h_s, is a constant. We now have a linearized equation:

$$A\frac{dh}{dt} = q_{in}(t) - \beta\left[h_s^{1/2} + \left(\frac{1}{2}h_s^{-1/2}\right)(h - h_s)\right]$$

Define deviation variables. Here they are shown with a superscript delta:

DEFINE DEVIATION VARIABLES ($^{\Delta}$)

$$h^{\Delta} = h - h_s \Rightarrow h = h^{\Delta} + h_s \Rightarrow \frac{dh}{dt} = \frac{dh^{\Delta}}{dt} + \cancel{\frac{dh_s}{dt}}^{\,0}$$

$$q_{in}^{\Delta} = q_{in} - q_{in,s} \Rightarrow q_{in} = q_{in}^{\Delta} + q_{in,s}$$

AT STEADY STATE ($_s$)

$$A \frac{dh_s}{dt} = 0 = \varphi_{IN,s} - \beta h_s^{1/2}$$

SUBTRACT SS EQN FROM ABOVE

$$\frac{dh^\Delta}{dt} + \underbrace{\left(\frac{\beta}{2A h^{1/2}}\right)}_{\varphi} h^\Delta = \frac{\varphi_{IN}^\Delta(t)}{A}$$

The equation is linear in the deviation variables.

Remember that this linearized equation is an approximation. The error will increase as the absolute magnitude of the deviation in height increases.

We are going to use deviation variables in all of our problems, even those that we do not need to linearize.

Also, in all of our problems, we will specify that the system is initially at a steady state before any change in the input variables occur.

Oh yes, there is another little problem...

Even after we linearize our math model, we can get differential equations that are difficult to solve.

But we are going to get help from Pierre-Simon, marquis de Laplace (1749 - 1827).

Laplace devised an integral transform, called the Laplace transform:

$$\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt$$

$$= F(s)$$

The Laplace transform converts linear differential equations with constant coefficients into algebraic equations in terms of the transform variable s. For example, for a first derivative:

$$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0)$$

$$= sF(s) \quad \text{for } f(0) = 0$$

After applying the Laplace transform, we solve the algebraic equations for the unknowns, the values of the output variables.

This solution will be in the transform domain or the "frequency domain," so called because the transform variable s has the dimension of inverse time, i.e., frequency.

So, to finish the problem, we "back transform" or "inverse transform" the solution back into the time domain that we live in.

The good thing is, you have taken Math 20D, so you already know about the Laplace Transform.

The funny thing is, our systems are described by differential equations but most of our math work is going to be algebra.