Control of an inherently linear system - Part 1- Proportional action only

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Consider the tank in SimzLab, Control Lab, Division 1, Lab 6, Temperature Control
www.SimzLab.com

Use the following parameter values - these should be the default values set when you first enter the lab - if not, change the settings to these values:

$V = 0.1\ m^3$ = volume of liquid in tank $C_p = 2$ kJ/kg/K = liquid heat capacity
$q = 0.005\ m^3/s$ = liquid flow rate $\rho = 1000\ kg/m^3$ = liquid density
UA = 10 kJ/min/K = heat transfer coefficient U times heat transfer surface area A

Use feedback control with only the proportional mode active. As you enter the lab, click the I and D tabs and uncheck the boxes so that you deactivate the integral and derivative modes.

When you first enter the lab, the proportional gain should be set to 3.0. This number is dimensionless, since (% per %) is dimensionless. Below $K_c = 3.0$ and not 3.0 %.

Change the set point temperature to 350 K.

For these parameter values, proportional-only control, $T_{in} = 300$ K, and a set point temperature $T_{sp} = 350$ K, we see that the steady-state temperature in the tank is T = 330 K and the jacket temperature $T_j = 360$ K. We can see that there is a steady-state error or "proportional offset" of 20 K.

The energy balance for the open-loop process (the tank without the controller) is:

$$\rho C_p V \frac{dT}{dt} = UA(T_j - T) + \rho C_p q (T_{in} - T)$$

$$\tau_P \frac{dT}{dt} + T = \left(\frac{\tau_P}{\tau_x}\right) T_j + \left(\frac{\tau_P}{\tau_f}\right) T_{in}$$

$$\tau_P \frac{dT}{dt} + T = K_P T_j + K_L T_{in}$$

where

$\tau_f = \left(\frac{V}{q}\right)$ = time constant for fluid flow = 20 min for the parameter values set here,

$\tau_x = \left(\frac{\rho C_p V}{UA}\right)$ = time constant for heat exchange = 20 min,

$$\tau_P = \left[\frac{1}{\frac{1}{\tau_f} + \frac{1}{\tau_x}} \right]$$ = time constant for the overall open-loop process = 10 min

$$K_P = \left(\frac{\tau_P}{\tau_x} \right)$$ = open-loop gain for change in manipulated variable T_j, here $K_p = 0.5$

$$K_L = \left(\frac{\tau_P}{\tau_f} \right)$$ = open-loop gain for change in load or disturbance variable T_{in}, here $K_L = 0.5$

With deviation variables about the initial steady state, the equation has the same form because the system is inherently linear.

$$\tau_P \frac{dT^\Delta}{dt} + T^\Delta = K_P T_j^\Delta + K_L T_{in}^\Delta$$

Apply the Laplace transform to get the transfer function equation for the open-loop process:

$$T(s) = G_P(s) T_j(s) + G_L(s) T_{in}(s)$$

$$G_p(s) = \frac{K_P}{(\tau_P s + 1)} = \frac{0.5}{(10 s + 1)}$$ for the parameter values set here, and

$$G_L(s) = \frac{K_L}{(\tau_P s + 1)} = \frac{0.5}{(10 s + 1)}$$

Now add the feedback controller to the transfer function equation.

$$T(s) = G_1(s) R(s) + G_2(s) T_{in}(s)$$

where *R(s)* is the transform of the set point (deviation) for the tank temperature, and where

$$G_1(s) = \frac{G_{sp} G_c G_a G_p}{1 + G_c G_a G_p G_m} = \frac{K_c G_p}{1 + K_c G_p}$$

$$G_2(s) = \frac{G_L}{1 + G_c G_a G_p G_m} = \frac{G_L}{1 + K_c G_p}$$

$G_{sp} = G_a = G_m = 1$ are specified for this system,

For other systems, the actuator and measurement devices may have dynamic responses of their own such that their G's may be functions of the transform variable *s* and not constant gains as shown here.

$G_c = K_c$ for proportional mode control only

Now go back and look at the simulator. At the parameter settings listed above, the set point is 350 K and the tank temperature is 330 K, so this proportional controller is giving us a steady-state error or "proportional offset" of 20 K. Let's check this value with the equations.

The closed-loop transfer function equation that we derived gives deviations from the initial steady state. It will not give us the value of the proportional offset at the initial steady state. To get that value, we have to look at the system equations in the time-domain.

$$T = K_P T_j + K_L T_{in}$$

The jacket temperature is determined by the proportional controller:

$$T_j = 300 + K_c (T_{sp} - T)$$

Solving for the steady-state temperature of the liquid in the tank:

$$T = K_P (300 + K_c (T_{sp} - T)) + K_L T_{in}$$

$$T = \frac{K_P (300 + K_c T_{sp}) + K_L T_{in}}{1 + K_P K_c}$$

For the parameter values above, T = 330 K, which is the temperature obtained in the simulator.

For unit step inputs, the steady-state gains are obtained by setting s = 0 in the transfer functions, from the final value theorem. Here we are using $K_c = 3.0$.

For the servo problem (unit step change in set point), set s = 0 in G_1 and find the steady-state gain is 0.6 K. For a $K_c = 1.0$, the gain is 0.33. Increasing the proportional gain K_c increases the steady-state gain for the servo problem. That is good. A gain of 1 K is desirable for this servo problem.

For the load problem (unit step change in T_{in}), set s = 0 in G_P and find the steady-state gain is 0.2 K. For a $K_c = 1.0$, the gain is 0.33 as we determined in class. Increasing the proportional gain K_c decreases the steady-state gain for the load problem. That is good. A gain of 0 K is desirable for this load problem.

Do experiments in the simulator to confirm these results. Rather than increasing T_{in} or the set point by 1 K as in a unit step input, increasing them by 20 K so that you can get a bigger change and, thus, more accurately determine the gain values. Since the system is linear, a change in load T_{in} of 20 K should result in a change in the tank T of 20(0.2 K) = 4 K for a proportional gain of $K_c = 3.0$.

Let's do some algebra on the load transfer function G_2:

$$G_2(s) = \frac{G_L}{1 + K_c G_p} = \frac{K_L / K_2}{(\tau_P / K_2) s + 1}$$

where

$$K_2 = 1 + K_c K_P$$

This is a linear, first-order system to which we have added proportional control. The closed-loop system remains a first-order system. This will change as we add in the other control modes.

For this control system, the closed-loop time constant is (τ_P / K_2). The same result is obtained for G_l since the denominators are the same.

As the proportional gain K_c increases, the closed-loop time constant decreases. That means the system responds faster as we increase the proportional gain.

The open-loop time constant for the process is τ_P. Since K_2 is greater than 1, the closed-loop time constant is smaller than the open-loop time constant. That is, the closed-loop system responds faster than the open-loop system. This is expected since the control system is acting on the process.

If increasing the proportional gain moves the steady-state gains in the right direction and speeds up the response, then why don't we just make the proportional gain value very large? One thing that prevents this is that actual physical devices have limits to their responses that we don't see if we only look at the basic theoretical equations. For instance, a real actuator can only change between physical minimum and maximum limits. For example, in the simulator, the jacket temperature can only be set to a maximum value of 400 K. A very large proportional gain setting in our equations could, theoretically, require the jacket temperature to exceed that physical limit.

Note in the simulator that there is a "manual bias" that can be set. Go back to these settings: T_{in} = 300 K, K_c = 3.0. Change the set point temperature to 320 K. Set the manual bias to 40 % such that T_j = 340 K. The tank temperature now is 320 K, the same as the set point. Using the manual bias, we can compensate for the proportional offset but only at a constant, steady state.

Now change T_{in} to 320 K, a change of 20 K. Above we predicted that the steady-state gain for a unit step load change is 0.2 K. For a load change of 20 times that, we expect a steady-state gain of 4 K, and that is what we see. But we want a steady-state gain for a load change to be 0 K. So a manual bias doesn't eliminate the proportional offset except at the original settings.

In order to eliminate the steady-state error under all conditions, we are going to have to add in the integral mode of control.

Also read Example 5.1, p. 97, in Chau's textbook.