"Real" PD controller based on a "lead-lag" element

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See Chau's textbook, p. 88, Eqn 5.7, Gc = Kc*(tauD*s + 1) / (alpha*tauD*s + 1)

An ideal PD controller has Gc = Kc*(1 + tauD*s). One trouble with this is that you can't use Matlab's control toolbox functions (step, impulse, lsim) to plot this controller's response because its numerator (= 1 + tauD*s, order 1) is higher-order than its denominator (= 1, order 0).

Also, this type of controller can't be perfectly implemented in real life because you can't instantaneously determine the rate of change of a signal; you need some finite time over which to measure the signal before you can determine its rate of change.

This can be fixed mathematically by putting the controller signal through a first-order "lag element." For we chemical engineers, it's sort of like putting the controller signal through a mixing tank before sending it on to the actuator.

The "lead" element in "lead-lag" is the tauD*s term in the numerator. The "lag element" is the (alpha*tauD*s + 1) term in the denominator.

Now, with the first-order lag added, we can use Matlab to explore the effect of the various terms. In the screen shots below, Kc = 1, and tauD = 1.

The input is the grey curve, which is a sine wave with amplitude 1 in the plots. The output is the blue curve. The plot on the left below is the "real" PD controller with alpha = 0.1 using the full Eqn. 5.7.

With derivative mode active note that the peaks in the output (blue) occur earlier (lead) the input peaks.

The plot on the right is the proportional-mode contribution with the lag element, and with alpha = 0.1. Gc = Kc*1 / (alpha*tauD*s + 1). We can see that the lag doesn't affect the proportional contribution significantly with this alpha value.
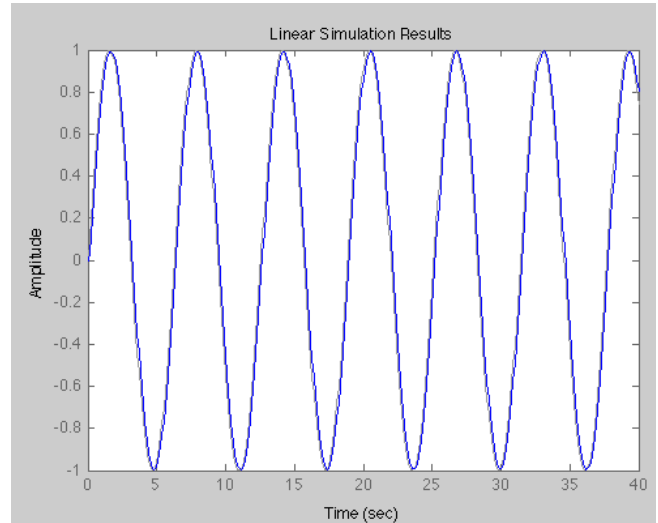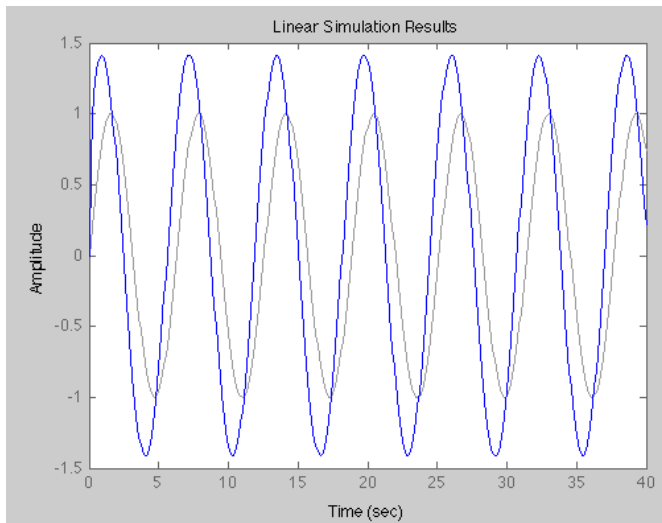
The plot on the left below is the derivative mode contribution with the lag element with alpha = 0.1.
Gc = Kc*tauD*s / (alpha*tauD*s + 1). Note that the output magnitude for both the proportional mode and derivative mode is about 1, however, the output magnitude of the PD controller is approximately 1.4. The P and D outputs can be summed to get the PD output, but note that the P and D curves are phase shifted from each other, so their summed magnitude isn't approximately 2.

The plot on the right has alpha = 0.01. Look at the blue output curve at and just after time zero. With alpha this small, 0.01, the lag element has little effect other than allowing us to use Matlab's control tool box.
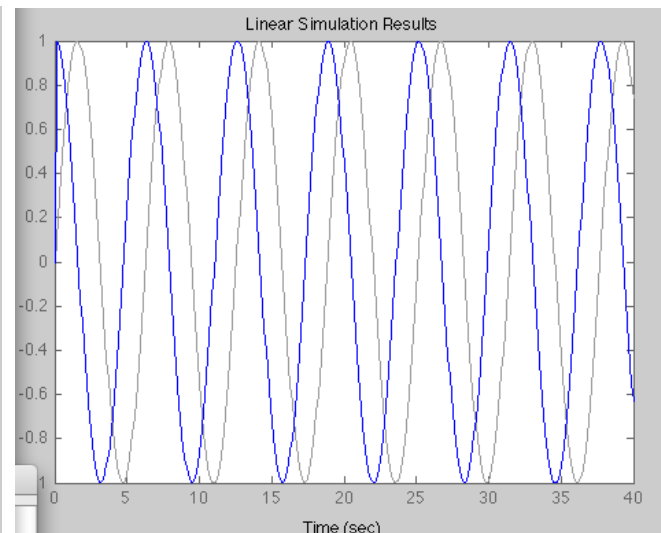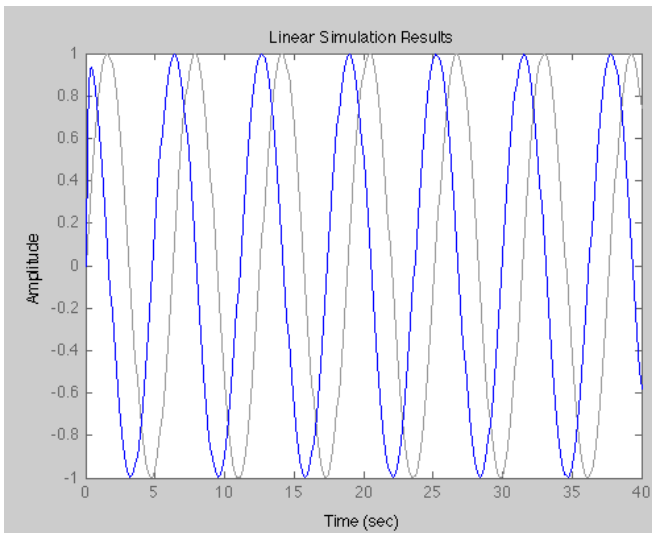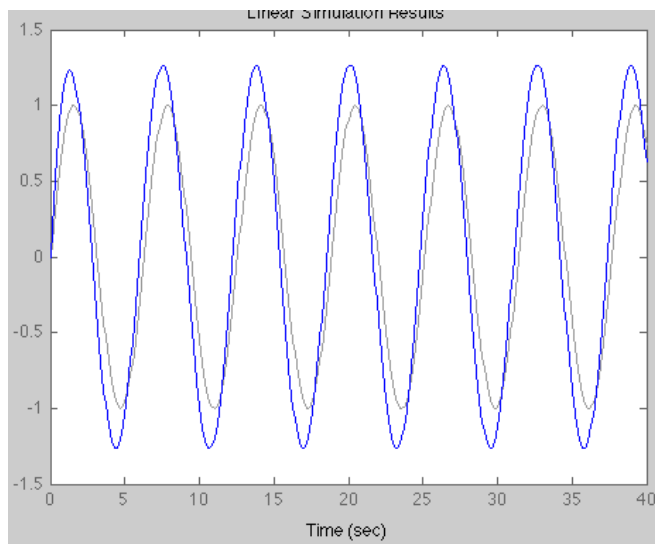

When alpha gets large it affects both the proportional and derivative modes, as expected.
The plot on the left below is the derivative-mode contribution with the lag element, and with alpha = 0.5.

The plot on the right below is the proportional-mode contribution with the lag element, and with alpha = 0.5.
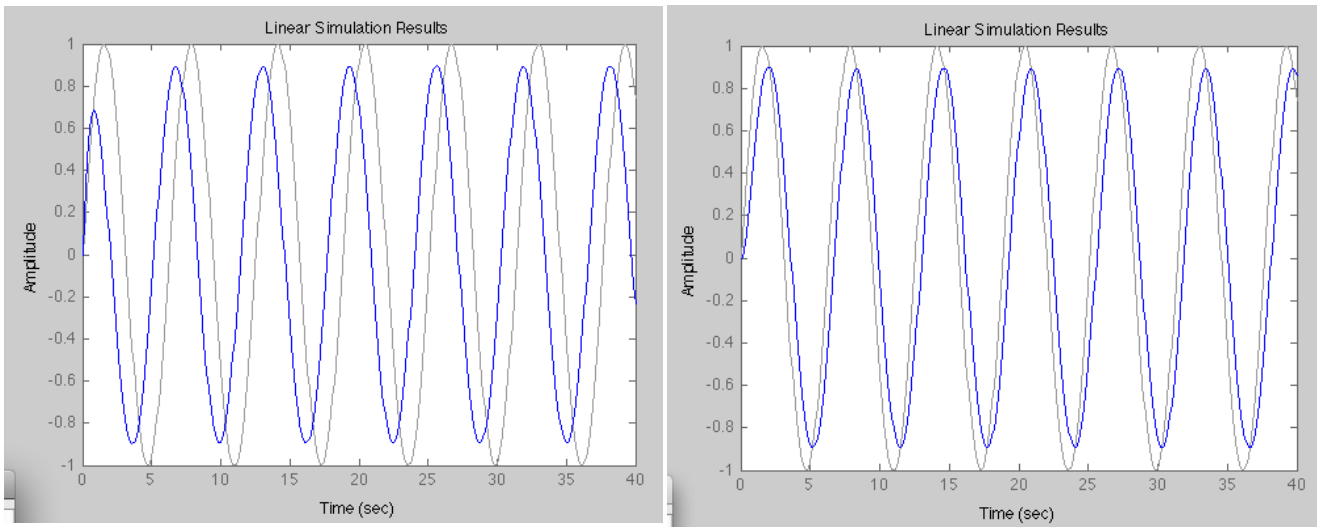
The final plot below is the "real" PD controller with alpha = 0.5.

Chau says the typical values of alpha are between 0.05 and 0.2. We have used a wider range here to see the effect of alpha more clearly.